Inflation And Quintessence In String Theory

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inflation

- Inflation is a period of accelerated expansion of the universe, taken place about 10^{-34} seconds after Big Bang.
- It provides a beautiful mechanism to generate the observed CMB anisotropy and distrbution of large scale structure.
- Inflation involves dynamical evolution of scalar field (so that $\omega = p/\rho$ is slightly greater than 1), coupled to gravity, that was at one time displaced from the minimum of its potential and it slowly rolls down towards the minimum.
- During this slow roll, horizon distance grows nearly exponentially relative to Hubble radius.
- At the end of slow roll (end of inflation) they differ by e^N, $N = \int_{t_i}^{t_e} H dt$ is called number of e-folds.

- As the field approaches and inevitably overshoots the minimum, it oscillates around the minimum on a time scale short compared to Hubble time (determined by the curvature of potential).
- The field decays to other lighter fields (to which it is couples), dampening the oscillations.
- As the decay product thermalize, the universe is reheated. The highly non-adiabatic reheating process increases the entropy within the inflating patch by many ordrers of magnitude.
- Thus "Inflation" should include Reheating.

connecting to observations

- Our present understanding of the origin of the structure of the universe is that it originated from small "seed" perturbations which grew to all the structures we observe.
- When the universe became matter dominated (around 1000 yrs after BB) primordial density inhomogeneities ($\frac{\delta\rho}{\rho} \sim 10^{-5}$) get amplified by gravity.
- Existence of these small inhomogeneities are the COBE anisotropies ($< \frac{\delta T}{T}, \frac{\delta T}{T} >$).
- Density perturbations are due to quantum fluctuations of the scalar field in de Sitter space.
- Note that $\delta\phi \Rightarrow \delta T_{\mu\nu} \Rightarrow \delta g_{\mu\nu}$.
- This leads to (1) scalar perturbation or density perturbation and (2) tensor perturbation or curvature perturbation.
- Observational data puts bound on both.

connecting to observations

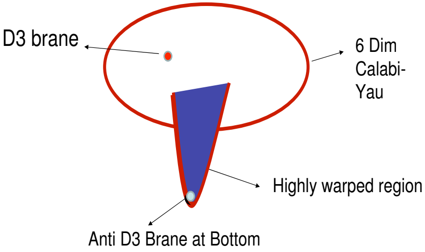
- COBE observations imply scale-invariant and Gaussian fluctuation in Temp.
- Constraints on amplitude of density perturbation: $\delta_H = \frac{2}{5} P_R^{1/2} = \frac{1}{5\pi\sqrt{3}} \frac{V^{3/2}}{M_{pl}^3 V'} = 1.91 \times 10^{-5}$.
- P_R is the power spectrum computed in terms of two point correlation function of the density perturbation. Numerical value is from COBE result.
- An effective spectral index $n(k)$ is defined by $n(k) - 1 \equiv \frac{d \ln P_R}{d \ln k}$ to study the scale dependence of the power spectrum. This is same as the assumed power law behaviour of $P_R \sim k^{n-1}$ over an interval of k when $n(k)$ is constant.
- These are related to the slow roll parameters : $n - 1 \simeq 2\eta - 6\epsilon$, similarly $n_{grav} = -2\epsilon$ and others.

Planck Sensitivity

- Inflation is sensitive to Planck scale Physics: Planck-suppressed operators generically make critical contributions to the dynamics.
- This provides remarkable oppertunity to probe aspects of ultraviolet completion of gravity (String Theory) through observations. To make meanigful use of this connection we should
- compute Planck-suppressed contributions to inflation action in String Theory
- look for what kind of inflation is natural in String Theory
- It turns out that for small inflaton excursions, $\Delta\phi \leq M_{Pl}$ one must control corrections with $\mathcal{O}_\Delta$ with $\Delta \leq 6$ and for large field excursions $\Delta\phi >> M_{Pl}$ we need to control infinite series of corrections with arbitrary large Δ.

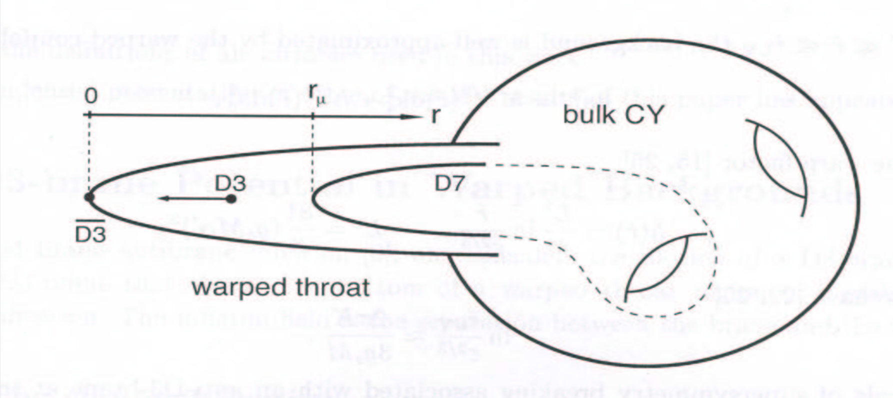
- We should carefully examine Planck-suppressed contributions to inflaton potential in String theory.
- Options: (1) Invoke a symmetry to forbid all such contributions
- (2) Enumurate all relevant contributions and deternine whether a fine-tuned inflation can occur i.e. arrange for cancellations.
- Contributions to inflaton potential arise from integrating out massive fields. In String Theory massive fields include the stabilized moduli.
- Infact option (2) for the brane inflation proposal of KKLMMT following the flux compactification and moduli stabilization a la KKLT showed that the inflaton mass becomes in order of the Hubble constant.
- This led to failure of slow roll inflation in this model.

KKLMMT framework

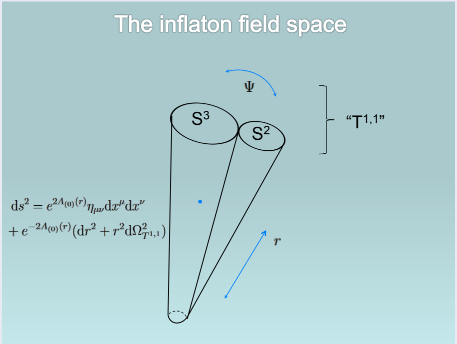


The KKLMMT Model

- New proposal like forcing the D3- brane to move deep inside the throat did not yield satisfatory result (BDKMS 2007; SP, Tsujikawa, Sami 2007).



Proposal of BDKKM 2008, 2009, 2010, compactification effects in UV



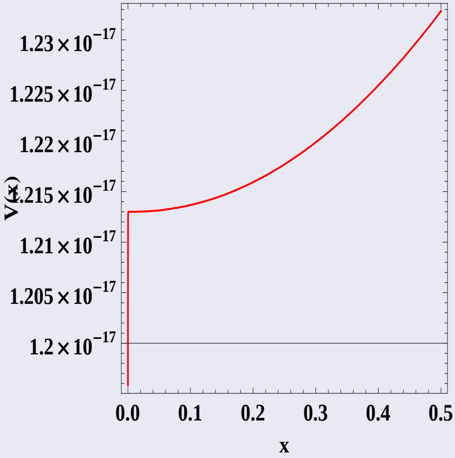
- Compute the Inflaton potential taking the contributions from Coulomb Interaction, curvature couplings, couplings to moduli in Kahler and in Super potential.

- Idea: Classify all possible corrections starting with infinite throat geometry and study all perturbations in UV consistent with sugra eqn of motion. Check which of these affect inflaton potential. Then classify possible corrections that can appear in true compactification.
- Tool: Ads/CFT allows to write arbitrary contribution to Inflaton Lagrangian, encompassing all above effects, in terms of supergravity contributions.
- From this most general solution one can read off the potential taking the help of matching between sugra G-flux modes and CFT operators cf Ceresole, D all'Agata, D' Auria 1999)

Result

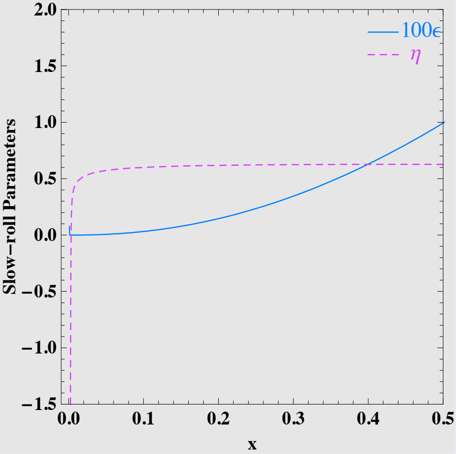
- The potential $V(\phi, \Psi) = V_0 + \sum_i b_i \phi^{\Delta_i} h_i(\Psi)$
- 724 terms for $\Delta < 4$!
- Inflation dynamics involves evolution of all 6 fields
- Single field Inflation by taking Ψ arbitrary constants Ali, Chingagbam, Panda, Sami 2008; Ali, Deashamukhya, Panda, Sami 2010
- Since small field inflation, kept only few terms such that we have flat potential satisfying slow-roll.
- $V = V_0 + C_{3/2}\phi^{3/2}$ turns out to satisfy all the ingradients we need to satisfy observational constraints but for highly fine tuned value of $C_{3/2}$ and initial value of inflaton..
- If we keep terms upto $\Delta = 5/2$, fine tuning reduces by order of three and we can have a range of values for each of the parameters to yield a model satisfying all the observational constraints.

Potential

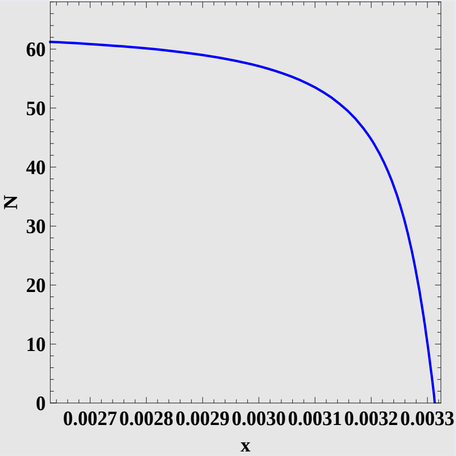


1.23×10^{-17}
1.225×10^{-17}
1.22×10^{-17}
1.215×10^{-17}
1.21×10^{-17}
1.205×10^{-17}
1.2×10^{-17}
V(x)
0.0
0.1
0.2
0.3
0.4
0.5
x

Slow roll parameters

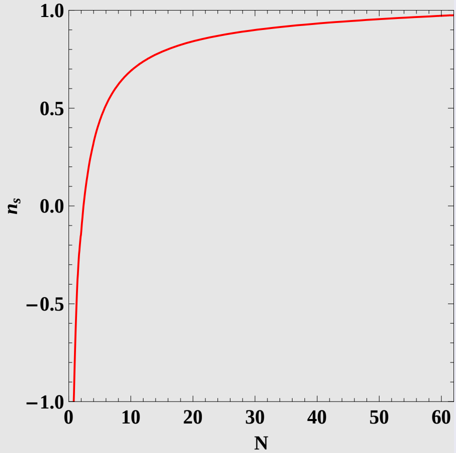


e-folding



60
50
40
30
20
10
0
N
0.0027
0.0028
0.0029
0.0030
0.0031
0.0032
0.0033
x

spectral index



Reheating

- The effective potential here belongs to the class of non-oscillatory inflation models .
- Conventional reheating does not work, try Instant preheating (Felder, Kofman, Linde)
- Inflaton interacts with another scalar field which has a Yukawa type interaction with a Fermi field:
- $L_{int} = -1/2g^2(\phi - v)^2\chi^2 - h\bar{\psi}\psi\chi$, $v = \phi_{end}$, for χ to be light.
- Preheating commences when effective mass $m_\chi = g|\phi - v|$ starts changing non-adiabatically:
- $|\dot{m}_\chi| \gtrsim m_\chi^2$ or $|\dot{\phi}| \gtrsim g(\phi - v)^2$
- Condition for particle production satisfied if
- $|\phi| \lesssim |\phi_{prod}| = v - \sqrt{\frac{|\dot{\phi}_{prod}|}{g}}$

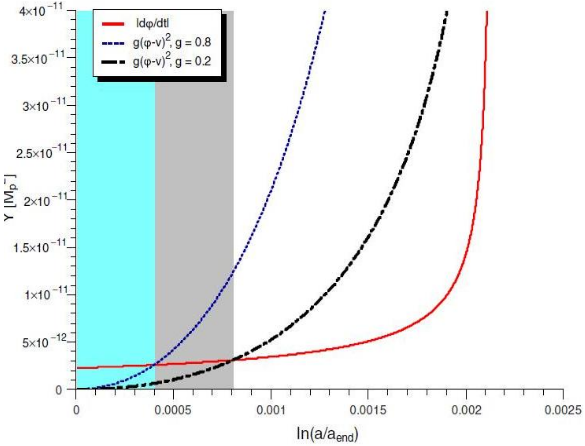


Figure: Post-inflationary evolution of $g(\phi - v)^2$ and $|\dot{\phi}|$, Violation of adiabatic condition takes place in the shaded region.

- We find for generic values of g preheating takes place instantaneously.
- Lower limit on g is fixed such that non-adiabatic process ends before we reach singularity.
- Momentum of particles is estimated by $k_p \sim \delta t^{-1} \sim \frac{|\dot{\phi}|}{|\phi|}$
- Occupation number (finite value during δt) : $N_k \sim \exp\left(-\pi k^2/k_p^2\right)$
- Number density : $n_\chi \simeq k_p^3$ which leads to $\rho_\chi = m_\chi n_\chi (\frac{a_{end}}{a})^3$.

- Assuming that the energy of χ particles, produced after inflation, is thermalized instantaneously giving rise to radiation energy density:
- $\rho_\chi \;=\; \rho_r \Rightarrow$
- Reheating temperature T: $10^{10}GeV \lesssim T \lesssim 10^{11}GeV$ for permissible values of range of g.
- Decay rate of χ particles to fermions : $\Gamma_{\bar{\psi}\psi} = h^2 m_\chi/8\pi$ must be larger than the expansion rate of the universe, to make the back reaction of χ particles on the background evolution negligible $\Rightarrow$ Lower bound on h.
- This, along with generic values of parameters g leads to the region in parameter space (h, g) (for $\alpha = 0.5$):

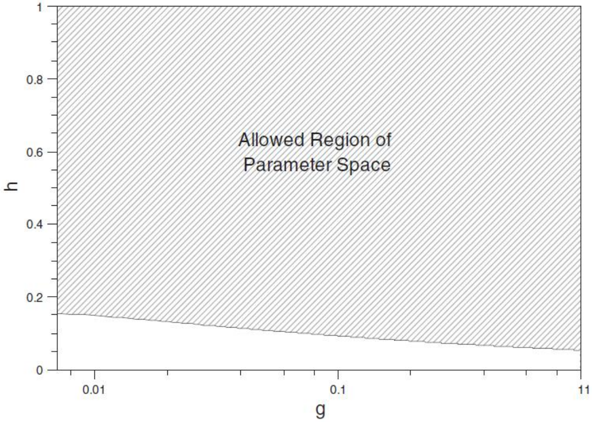


Figure: shaded regions $0.006 \lesssim g \lesssim 1$ and $0.053 \lesssim h \lesssim 1$ lead to successful instant heating,

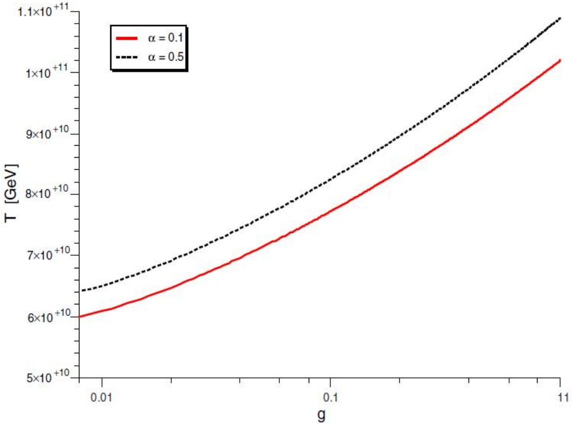


Figure: Reheating temp vs g; $T \backsimeq \rho_\chi^{1/4}$ varies between $6 \times 10^{10} - 1.1 \times 10^{11}$ Gev

- To conclude, there is fair control over Brane inflation model building consistent with observations and a reasonable understanding of reheating the universe.
- Inflation from Axion Monodromy is a large field Inflation model aimed at bounds on gravity waves in future experiments.
- We, being satisfied with brane inflation model, aim at exploiting the idea of Axion monodromy to build a model for Time varying vacuum energy called Quintessence.
- This is responsible for the late time accelerated expansion as opposed to constant vac energy/Dark energy with $\omega = -1$.
- Observations in next decade will determine ω and thus will decide one of the two.

Axion monodromy Quintessence Panda, Sumimoto, Trivedi

- Idea behind quintessence is quite similar to that of Inflation i.e. slowly rolling scalar field.
- But the energy scale (Λ) of quitessence is of order 10^{-3} eV, much smaller than supersymmetry breaking scale
- How do we get a potential that meets slow roll conditions despite the high scale of SUSY breaking?
- This reqires mass of Q-field is order of H today ($\sim 10^{-33}$ eV) and thus is much smaller than M_{SB} ($\sim 1 - 10$ TeV or much higher).
- But scalar fields are known to have their mass driven up to M_{SB} !

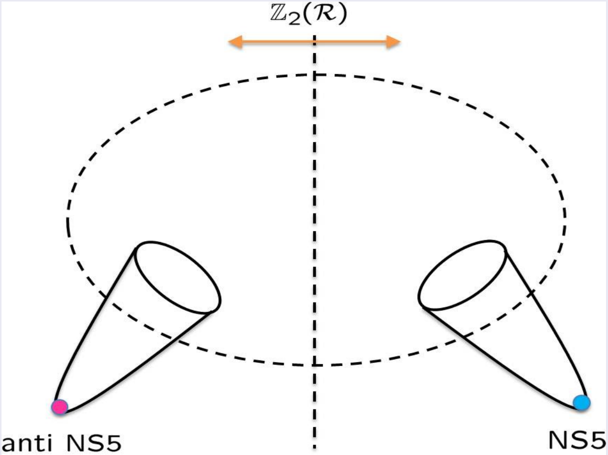
- Ensuring $m \sim H$ is a challange.
- We also need full UV theory to suppress the higher dimensional operators which can contribute to the mass (need of String theory).
- In the model we construct , the Q-field undergoes an excursion of order Planck scale during its evolution and the potential has to be flat for the whole range in field space (dificult in EFT).
- We can overcome all the above problems if we consider the Q-field to be the axion field arising from RR sector in compactification of Type II B string theory .
- The shift symmetry associated with the axion field is broken in the presence of NS5-branes placed in highly warped regions of compact space .
- This is the idea of Axion monodromy used to coonstruct large field inflation (Silverstein, Westpal; MSW).

Sample analysis for Axion Quintessence

- Consider a potential, linear in axion: $V(\phi) = \mu^4 a = \frac{\mu^4}{f_a}\phi$, μ is a mass scale, ϕ canonically normalized field and a is dim less axion.
- Slow-roll conditions say: $phi > M_{Pl}$. i.e. $a > \frac{M_{Pl}}{f_a}$
- Potential energy in the axion today is of order of total energy density in the universe i.e. $\mu^4 a \sim \Lambda^4 \sim 10^{-12}\,(\mathrm{eV})^4$
- Thus condition scale μ is $\mu^4 \lesssim \frac{f_a}{M_{Pl}}\Lambda^4$
- Computing the eqn of state parameter ω and comparing with the observational data reveals that $\phi \sim 2.14 M_{Pl}$
- From the eqn of motion for the axion field one can estimate the change in axion field during the evolution of the universe , upto current epoch: $\delta\phi \sim \frac{M_{Pl}^2}{\phi}$
- For ϕ of order Planck scale, the change of ϕ during evolution is also of Planck scale!

- Thus the potential must be slowly varying for the entire range of evolution of the axion field: significant constraint to meet
- We work with flux compactification in IIB theory and consider the axion to come from C_2.
- Ten-dim action :$S = \frac{1}{(2\pi)^7\alpha'^4}\int d^{10}x\sqrt{-g}\left[\frac{1}{g_s^2}R - \frac{1}{12}\partial_\mu C_{ab}\partial^\mu C_{a'b'}g^{aa'}g^{bb'} + \cdots\right]$ give to Four-dim action:
  $S = \int d^4x\sqrt{-g_4}\left[\frac{M_{Pl}^2}{2}R - f_a^2\frac{1}{2}(\partial a)^2\right]$
- Four-dim Planck scale :$M_{Pl}^2 = \frac{2L^6}{(2\pi)^7 g_s^2\alpha'}$; $V = L^6(\alpha')^3$ is volume of internal space with L dim-less modulus and $f_a^2 \sim \frac{g_s^2 M_{Pl}^2}{L^4}$.
- Above f_a and the slow-roll condition imply axion has to satisfy $a > \frac{L^2}{g_s}$ which will help us to obtain a linear potential.

Our mechanism



- The shift symmetry is broken due to presence of a pair of NS5-brane and $\overline{\mathrm{NS5}}$-brane (needed for charge conservation).
- The axion, which is the zero mode of the RR two-form C_2 that arises due to a non-trivial two-cycle Σ, induces a D3-brane charge on the 5-brane which wraps the two-cycle Σ. cf WZ term $\int C_2 \wedge C_4$, $a\alpha' = \int_\Sigma C_2$ It also induces $\overline{D3}$-brane charge on the anti 5-brane.
- This results in additional 3-brane tension being induced on both the 5 and anti 5-branes which depends on the axion. The resulting potential turns out to be approximately linear in the axion, can be read out from DBI action: $V = 2\epsilon \frac{1}{(2\pi)^5 g_s^2 \alpha'^2} \sqrt{L^4 + g_s^2 a^2} \rightarrow V = 2\epsilon \frac{1}{(2\pi)^5 g_s \alpha'^2} a$
- $\epsilon = e^{4A_0}$ where the warp factor at the location of the 5-brane is e^{A_0}.

- We see that the mass scale is $\mu^4 = \epsilon \frac{2}{(2\pi)^5 g_s \alpha'^2}$ i.e.$\mu^4 \propto e^{4A_0}$.
- When $a \gg \frac{L^2}{g_s}$ is met the 5-brane tension itself is insignificant compared to the contribution due to the 3-brane charge.
- One can think of the setup as essentially consisting of a stack of 3-branes in one throat with another stack of anti 3-branes in the $\mathbb{Z}_2$ image throat.
- As axion evolves the induced 3-brane charge decreases and along with it the induced tension of the 3-branes also decreases.
- Slow-roll condition is already met. In addition if the warp factor is small enough the axion energy density $\rho \sim \epsilon \frac{2}{(2\pi)^5 g_s \alpha'^2} a \sim \Lambda^4$ can also be of the required small value.

- With little algebra, using the expression for four-dim Planck mass and expressing α'^2 in terms of M_{Pl} and the moduli L, g_s we have $\epsilon \sim \frac{2L^{12}}{(2\pi)^9 a g_s^3}\left(\frac{\Lambda}{M_{Pl}}\right)^4$
- Taking $L = 10, g_s = 1$ and $a \sim L^2$ and with $(\Lambda/M_{Pl})^4 \sim 10^{-123}$ we get $\epsilon \sim 10^{-120}$
- This is a small number but remember that ϵ is determined by the warp factor at the bottom of the throat which is exponentially sensitive to fluxes.
- Thus small ratios in flux can give the required large hierarchy between the energy density in the quintessence field and the Planck scale and second condition can be met taking reasonable values of L, g_s.
- Thus, we see that breaking the shift symmetry by placing 5-branes in highly warped throats allows us to meet the requirements for a model of quintessence.

Contributions to Potential from Moduli Stabilization and other correction

- Superpotential: If non-perturbative effects came from Euclidean D3-branes, we get contributions which are not suppressed.
- But if it came wrapped 7-branes the correction is exponentially suppressed.
- Kahler potential: There is correction$\sim b^2$ if the axion is a B_2 zero mode but no corrections for axion coming from C_2, hence we have taken axion from RR sector.
- Warping effect: The extra 3-brane charge which arose due to presence of axion gives additional warping which changes overall volume of internal space. Since volume was already stabilized, this change comes at the cost of energy which depends on axion. Same is the case for anti-3-brane.

- The leading contribution to potential cancels only if there is a $\mathbb{Z}_2$ symmetry $\mathcal{R}$ of the Calabi-Yau space, under which the two throats are exchanged,
- The sub-leading term does not cancel. However, it is linear in axion and does not spoil our proposal and we add it to the earlier linear potential and the combined potential is our $V = \mu^4 a$
- In summary, we have a reasonable model of Quintessence with linear potential derived from String theory

Thank You